A

An Introduction to Prolog

A.1 A Short Background

Prolog was designed in the 1970s by Alain Colmerauer and a team of researchers with the idea – new at that time – that it was possible to use logic to represent knowledge and to write programs. More precisely, Prolog uses a subset of predicate logic and draws its structure from theoretical works of earlier logicians such as Herbrand (1930) and Robinson (1965) on the automation of theorem proving.

Prolog was originally intended for the writing of natural language processing applications. Because of its conciseness and simplicity, it became popular well beyond this domain and now has adepts in areas such as:

- Formal logic and associated forms of programming
- Reasoning modeling
- Database programming
- Planning, and so on.

This chapter is a short review of Prolog. In-depth tutorials include: in English, Bratko (2001), Clocksin and Mellish (2003), Covington et al. (1997), Sterling and Shapiro (1994); in French, Giannesini et al. (1985); and in German, Bauman (1991). Boizumault (1988, 1993) contain a didactical implementation of Prolog in Lisp. Prolog foundations rest on first-order logic. Apt (1997), Burke and Foxley (1996), Delahaye (1986), and Lloyd (1987) examine theoretical links between this part of logic and Prolog.

Colmerauer started his work at the University of Montréal, and a first version of the language was implemented at the University of Marseilles in 1972. Colmerauer and Roussel (1996) tell the story of the birth of Prolog, including their try-and-fail experimentation to select tractable algorithms from the mass of results provided by research in logic.

In 1995, the International Organization for Standardization (ISO) published a standard on the Prolog programming language. Standard Prolog (Deransart et al. 1996) is becoming prevalent in the Prolog community and most of the available implementations now adopt it, either partly or fully. Unless specifically indicated, descriptions in this chapter conform to the ISO standard, and examples should run under any Standard Prolog implementation.

A.2 Basic Features of Prolog

A.2.1 Facts

Facts are statements that describe object properties or relations between objects. Let us imagine we want to encode that Ulysses, Penelope, Telemachus, Achilles, and others are characters of Homer's *Iliad* and *Odyssey*. This translates into Prolog facts ended with a period:

```
character(priam, iliad).
character(hecuba, iliad).
character(achilles, iliad).
character(agamemnon, iliad).
character(patroclus, iliad).
character(hector, iliad).
character(andromache, iliad).
character(rhesus, iliad).
character(ulysses, iliad).
character(menelaus, iliad).
character(helen, iliad).

character(ulysses, odyssey).
character(penelope, odyssey).
character(telemachus, odyssey).
character(laertes, odyssey).
character(nestor, odyssey).
character(menelaus, odyssey).
character(helen, odyssey).
character(hermione, odyssey).
```

Such a collection of facts, and later, of rules, makes up a **database**. It transcribes the knowledge of a particular situation into a logical format. Adding more facts to the database, we express other properties, such as the gender of characters:

```
% Male characters        % Female characters

male(priam).             female(hecuba).
male(achilles).          female(andromache).
male(agamemnon).         female(helen).
male(patroclus).         female(penelope).
male(hector).
male(rhesus).
male(ulysses).
male(menelaus).
male(telemachus).
male(laertes).
male(nestor).
```

or relationships between characters such as parentage:

```
% Fathers                  % Mothers
father(priam, hector).     mother(hecuba, hector).
father(laertes,ulysses). mother(penelope,telemachus).
father(atreus,menelaus). mother(helen, hermione).
father(menelaus, hermione).
father(ulysses, telemachus).
```

Finally, would we wish to describe kings of some cities and their parties, this would be done as:

```
king(ulysses, ithaca, achaean).
king(menelaus, sparta, achaean).
king(nestor, pylos, achaean).
king(agamemnon, argos, achaean).
king(priam, troy, trojan).
king(rhesus, thrace, trojan).
```

From these examples, we understand that the general form of a Prolog fact is: `relation(object1, object2, ..., objectn)`. Symbols or names representing objects, such as `ulysses` or `penelope`, are called **atoms**. Atoms are normally strings of letters, digits, or underscores "_", and begin with a lowercase letter. An atom can also be a string beginning with an uppercase letter or including white spaces, but it must be enclosed between quotes. Thus `'Ulysses'` or `'Pallas Athena'` are legal atoms.

In logic, the name of the symbolic `relation` is the **predicate**, the objects `object1`, `object2`, ..., `objectn` involved in the relation are the **arguments**, and the number `n` of the arguments is the **arity**. Traditionally, a Prolog predicate is indicated by its name and arity: `predicate/arity`, for example, `character/2`, `king/3`.

A.2.2 Terms

In Prolog, all forms of data are called **terms**. The constants, i.e., atoms or numbers, are terms. The fact `king(menelaus, sparta, achaean)` is a **compound term** or a **structure**, that is, a term composed of other terms – **subterms**. The arguments of this compound term are constants. They can also be other compound terms, as in

```
character(priam, iliad, king(troy, trojan)).
character(ulysses, iliad, king(ithaca, achaean)).
character(menelaus, iliad, king(sparta, achaean)).
```

where the arguments of the predicate `character/3` are two atoms and a compound term.

It is common to use trees to represent compound terms. The nodes of a tree are then equivalent to the functors of a term. Figure A.1 shows examples of this.

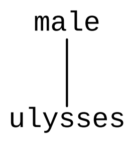
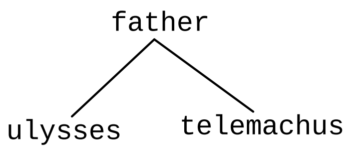
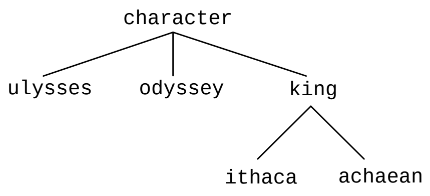
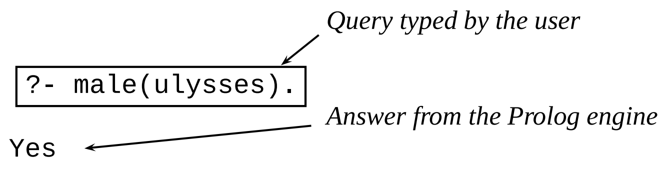
Terms	**Graphical representations**
`male(ulysses)`	
`father(ulysses, telemachus)`	
`character(ulysses, odyssey, king(ithaca, achaean))`	 character ulysses odyssey king ithaca achaean

Fig. A.1. Graphical representations of terms.

Syntactically, a compound term consists of a **functor** – the name of the relation – and arguments. The leftmost functor of a term is the **principal functor**. A same principal functor with a different arity corresponds to different predicates: `character/3` is thus different from `character/2`. A constant is a special case of a compound term with no arguments and an arity of 0. The constant `abc` can thus be referred to as `abc/0`.

A.2.3 Queries

A query is a request to prove or retrieve information from the database, for example, if a fact is true. Prolog answers yes if it can prove it, that is, here if the fact is in the database, or no if it cannot: if the fact is absent. The question *Is Ulysses a male*? corresponds to the query:



```
?- male(ulysses).
Yes
```

which has a positive answer. A same question with Penelope would give:

```
?- male(penelope).
No
```

because this fact is not in the database.

The expressions `male(ulysses)` or `male(penelope)` are **goals** to prove. The previous queries consisted of single goals. Some questions require more goals, such as *Is Menelaus a male and is he the king of Sparta and an Achaean?*, which translates into:

```
?- male(menelaus), king(menelaus, sparta, achaean).
Yes
```

where "`,`" is the conjunction operator. It indicates that Prolog has to prove both goals. The simple queries have one goal to prove, while the **compound queries** are a conjunction of two or more goals:

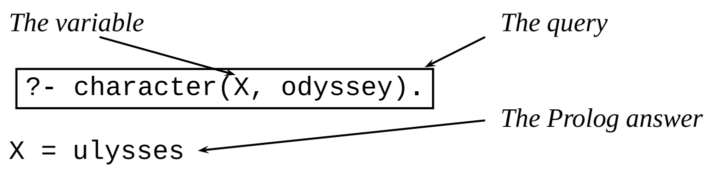
```
?- G1, G2, G3, ..., Gn.
```

Prolog proves the whole query by proving that all the goals `G1` ... `Gn` are true.

A.2.4 Logical Variables

The logical variables are the last kind of Prolog terms. Syntactically, variables begin with an uppercase letter, for example, `X`, `Xyz`, or an underscore "_". Logical variables stand for any term: constants, compound terms, and other variables. A term containing variables such as `character(X, Y)` can unify with a compatible fact, such as `character(penelope, odyssey)`, with the **substitutions** `X = penelope` and `Y = odyssey`.

When a query term contains variables, the Prolog resolution algorithm searches terms in the database that unify with it. It then substitutes the variables to the matching arguments. Variables enable users to ask questions such as *What are the characters of the Odyssey?*



```
?- character(X, odyssey).
X = ulysses
```

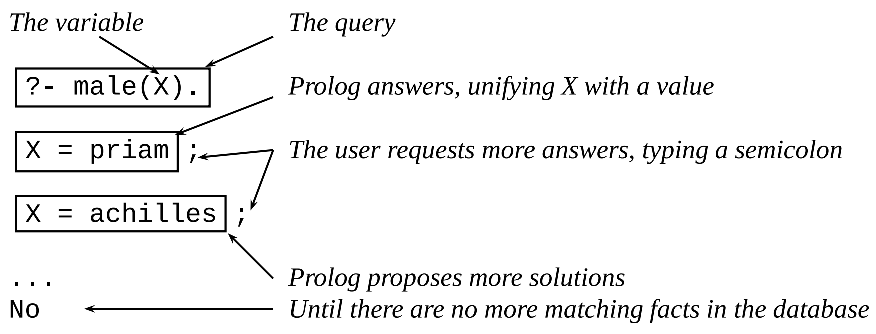
Or *What is the city and the party of king Menelaus?* etc.

```
?- king(menelaus, X, Y).
X = sparta, Y = achaean

?- character(menelaus, X, king(Y, Z)).
X = iliad, Y = sparta, Z = achaean

?- character(menelaus, X, Y).
X = iliad, Y = king(sparta, achaean)
```

When there are multiple solutions, Prolog considers the first fact to match the query in the database. The user can type "`;`" to get the next answers until there is no more solution. For example:


The variable
The query
Prolog answers, unifying X with a value
The user requests more answers, typing a semicolon
Prolog proposes more solutions
Until there are no more matching facts in the database

```
?- male(X).
X = priam ;
X = achilles ;
...
No
```

A.2.5 Shared Variables

Goals in a conjunctive query can share variables. This is useful to constrain arguments of different goals to have a same value. To express the question *Is the king of Ithaca also a father?* in Prolog, we use the conjunction of two goals `king(X, ithaca, Y)` and `father(X, Z)`, where the variable `X` is shared between goals:

```
?- king(X, ithaca, Y), father(X, Z).
X = ulysses, Y = achaean, Z = telemachus
```

In this query, we are not interested by the name of the child although Prolog responds with `Z = telemachus`. We can indicate to Prolog that we do not need to know the values of `Y and Z` using **anonymous variables**. We then replace `Y and Z` with the symbol “_”, which does not return any value:

```
?- king(X, ithaca, _), father(X, _).
X = ulysses
```

A.2.6 Data Types in Prolog

To sum up, every data object in Prolog is a term. Terms divide into atomic terms, variables, and compound terms (Fig. A.2).

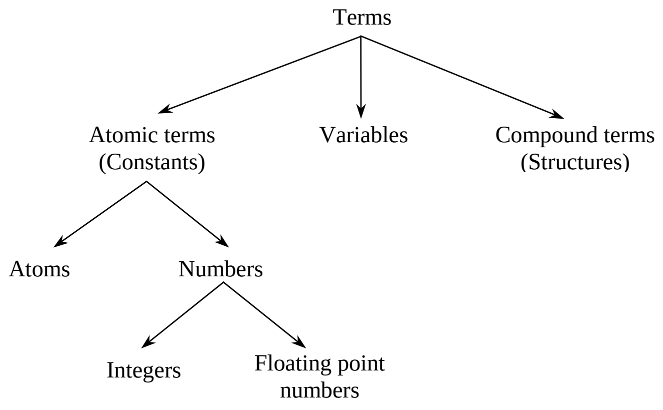


Fig. A.2. Kinds of terms in Prolog.

Syntax of terms may vary according to Prolog implementations. You should consult reference manuals for their specific details. Here is a list of simplified conventions from Standard Prolog (Deransart et al. 1996):

- Atoms are sequences of letters, numbers, and/or underscores beginning with a lowercase letter, as `ulysses`, `iSLanD3`, `king_of_Ithaca`.
- Some single symbols, called solo characters, are atoms: `! ;`
- Sequences consisting entirely of some specific symbols or graphic characters are atoms: `+ - * / ^ < = > ~ : . ? @ # $ & \ ‘`
- Any sequence of characters enclosed between single quotes is also an atom, as `'king of Ithaca'`. A quote within a quoted atom must be double quoted: `'I''m'`
- Numbers are either decimal integers, as `-19`, `1960`, octal integers when preceded by `0o`, as `0o56`, hexadecimal integers when preceded by `0x`, as `0xF4`, or binary integers when preceded by `0b`, as `0b101`.
- Floating-point numbers are digits with a decimal point, as `3.14`, `-1.5`. They may contain an exponent, as `23E-5` (23 10^{-5}) or `-2.3e5` (2.3 10^{-5}).
- The ASCII numeric value of a character `x` is denoted `0'x`, as `0'a` (97), `0'b` (98), etc.

- Variables are sequences of letters, numbers, and/or underscores beginning with an uppercase letter or the underscore character.
- Compound terms consist of a functor, which must be an atom, followed immediately by an opening parenthesis, a sequence of terms separated by commas, and a closing parenthesis.

Finally, Prolog uses two types of comments:

- Line comments go from the "`%`" symbol to the end of the line:
  ```
  % This is a comment
  ```
- Multiline comments begin with a "`/*`" and end with a "`*/`":
  ```
  /*
  this
  is
  a comment */
  ```

A.2.7 Rules

Rules enable to derive a new property or relation from a set of existing ones. For instance, the property of being the son of somebody corresponds to either the property of having a father and being a male, or having a mother and being a male. Accordingly, the Prolog predicate `son(X, Y)` corresponds either to conjunction `male(X), father(Y, X)`, or to `male(X), mother(Y, X)`. Being a son admits thus two definitions that are transcribed as two Prolog rules:

```
son(X, Y) :- father(Y, X), male(X).
son(X, Y) :- mother(Y, X), male(X).
```

More formally, rules consist of a term called the **head**, followed by symbol "`:-`", read if, and a conjunction of goals. They have the form:

```
HEAD :- G1, G2, G3, ... Gn.
```

where the conjunction of goals is the **body** of the rule. The head is true if the body is true. Variables of a rule are shared between the body and the head. Rules can be queried just like facts:

```
?- son(telemachus, Y).
Y = ulysses;
Y = penelope;
No
```

Rules are a flexible way to deduce new information from a set of facts. The `parent/2` predicate is another example of a family relationship that is easy to define using rules. Somebody is a parent if s/he is either a mother or a father:

```
parent(X, Y) :- mother(X, Y).
parent(X, Y) :- father(X, Y).
```

Rules can call other rules as with `grandparent/2`. A grandparent is the parent of a parent and is defined in Prolog as

```
grandparent(X, Y) :- parent(X, Z), parent(Z, Y).
```

where `Z` is an intermediate variable shared between goals. It enables us to find the link between the grandparent and the grandchild: a mother or a father.

We can generalize the `grandparent/2` predicate and write `ancestor/2`. We use two rules, one of them being recursive:

```
ancestor(X, Y) :- parent(X, Y).
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
```

This latter pattern is quite common of Prolog rules. One or more rules express a general case using recursion. Another set of rules or facts describes simpler conditions without recursion. They correspond to boundary cases and enable the recursion to terminate.

A query about the ancestors of Hermione yields:

```
?- ancestor(X, hermione).
X= menelaus;
X = helen;
X = atreus;
No
```

Facts and rules are also called **clauses**. A predicate is defined by a set of clauses with the same principal functor and arity. Facts are indeed special cases of rules: rules that are always true and `relation(X, Y)` is equivalent to `relation(X, Y) :- true`, where `true/0` is a built-in predicate that always succeeds. Most Prolog implementations require clauses of the same name and arity to be grouped together.

In the body of a rule, the comma "`,`" represents a conjunction of goals. It is also possible to use a disjunction with the operator "`;`". Thus:

```
A :-
  B
  ;
  C.
```

is equivalent to

```
A :- B.
A :- C.
```

However, "`;`" should be used scarcely because it impairs somewhat the legibility of clauses and programs. The latter form is generally better.

A.3 Running a Program

The set of facts and rules of a file makes up a **Prolog text** or program. To run it and use the information it contains, a Prolog system has to load the text and add it to the current database in memory. Once Prolog is launched, it displays a prompt symbol "`?-`" and accepts commands from the user.

Ways to load a program are specific to each Prolog implementation. A user should look them up in the reference manual because the current standard does not define them. There are, however, two commands drawn from the Edinburgh Prolog tradition (Pereira 1984) implemented in most systems: `consult/1` and `reconsult/1`.

The predicate `consult/1` loads a file given as an argument and adds all the clauses of the file to the current database in memory:

```
?- consult(file_name).
```

`file_name` must be an atom as, for example,

```
?- consult('odyssey.pl').
```

It is also possible to use the shortcut:

```
?- [file_name].
```

to load one file, for example,

```
?- ['odyssey.pl'].
```

or more files:

```
?- [file1, file2].
```

The predicate `reconsult/1` is a variation of `consult`. Usually, a programmer writes a program, loads it using `consult`, runs it, debugs it, modifies the program, and reloads the modified program until it is correct. While `consult` adds the modified clauses to the old ones in the database, `reconsult` updates the database instead. It loads the modified file and replaces clauses of existing predicates in the database by new clauses contained in the file. If a predicate is in the file and not in the database, `reconsult` simply adds its clauses. In some Prolog systems, `reconsult` does not exist, and `consult` discards existing clauses to replace them by the new definition from the loaded file. Once a file is loaded, the user can run queries.

The `listing/0` built-in predicate displays all the clauses in the database, and `listing/1`, the definition of a specific predicate. The `listing/1` argument format is either `Predicate` or `Predicate/Arity`:

```
?- listing(character/2).
character(priam, iliad).
character(hecuba, iliad).
character(achilles, iliad).
...
```

A program can also include directives, i.e., predicates to run at load time. A directive is a rule without a head: a term or a conjunction of terms with a "`:-`" symbol to its left-hand side:

```
:- predicates_to_execute.
```

Directives are run immediately as they are encountered. If a directive is to be executed once the program is completely loaded, it must occur at the end of the file.

Finally, `halt/0` quits Prolog.

A.4 Unification

A.4.1 Substitution and Instances

When Prolog answers a query made of a term T containing variables, it applies a **substitution**. This means that Prolog replaces variables in T by values so that it proves T to be true. The substitution `{X = ulysses, Y = odyssey}` is a solution to the query `character(X, Y)` because the fact `character(ulysses, odyssey)` is in the database. In the same vein, the substitution `{X = sparta, Y = achaean}` is a solution to the query `king(menelaus, X, Y)`.

More formally, a substitution is a set `{X1 = t1, X2 = t2, ..., Xn = tn}`, where `Xi` is a variable and `ti` is a term. Applying a substitution σ to a term T is denoted $T\sigma$ and corresponds to the replacement of all the occurrences of variable `Xi` with term `ti` in T for `i` ranging from `1` to `n`. Applying the (meaningless) substitution σ_1 = `{X = ulysses}` to the term `T1 = king(menelaus, X, Y)` yields `T1' = king(menelaus, ulysses, Y)`. Applying the substitution σ_2 = `{X = iliad, Y = king(sparta, achaean)}` to the term `T2 = character(menelaus, X, Y)` yields `T2' = character(menelaus, iliad, king(sparta, achaean))`.

A term T' resulting from a substitution $T\sigma$ is an **instance** of T. More generally, T' is an instance of T if there is a substitution so that $T' = T\sigma$. If T' is an instance of T, then T is **more general** than T'. Terms can be ordered according to possible compositions of instantiations. For example, `character(X, Y)` is more general than `character(ulysses, odyssey)`; `king(X, Y, Z)` is more general than `king(menelaus, Y, Z)`, which is more general than `king(menelaus, Y, achaean)`, which is itself more general than `king(menelaus, sparta, achaean)`.

A substitution mapping a set of variables onto another set of variables such as σ = {`X = A, Y = B`} onto term `character(X, Y)` is a **renaming substitution**. Initial and resulting terms `character(X, Y)` and `character(A, B)` are said to be **alphabetical variants**. Finally, a **ground** term is a term that contains no variable such as `king(menelaus, sparta, achaean)`.

A.4.2 Terms and Unification

To equate two terms, `T1` and `T2`, Prolog uses unification, which substitutes variables in the terms so that they are identical. Unification is a logical mechanism that carries out a two-way matching, from `T1` to `T2` and the reverse, and merges them into a common term. Prolog unifies terms to solve equations such as `T1 = T2`. It also uses unification in queries to match a goal or a subgoal to the head of the rule. Figure A.3 shows the intuitive unification of terms

```
T1 = character(ulysses, Z, king(ithaca, achaean))
```

and

```
T2 = character(ulysses, X, Y)
```

through a graphical superposition.

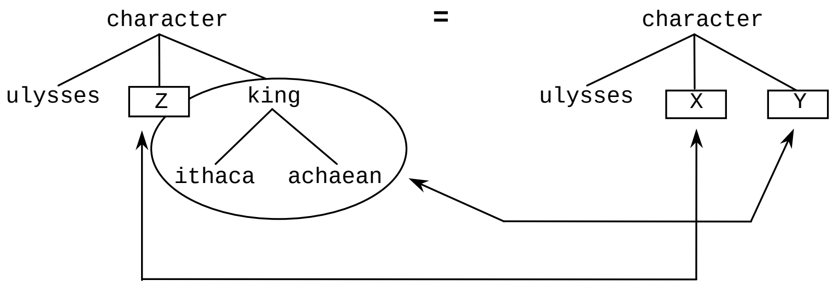


Fig. A.3. Unification of terms: a graphical interpretation.

The superposition of the two terms requires finding an instance common to both terms T_1 and T_2. This can be restated as there exist two substitutions σ_1 and σ_2 such that $T_1\sigma_1 = T_2\sigma_2$. A **unifier** is a substitution making T_1 and T_2 identical: $T_1\sigma = T_2\sigma$. In our example, there is an infinite number of possible unifiers. Candidates include the substitution σ = `{Z = c(a), X = c(a), Y = king(ithaca, achaean)}`, which yields the common instance: `character(ulysses,c(a), king(ithaca, achaean))`. They also include σ = `{Z = female, Z = female, Y = king(ithaca, achaean)}`, which yields another common instance: `character(ulysses, female, king(ithaca, achaean))`, etc.

Intuitively, these two previous unifiers are special cases of the unification of `T1` and `T2`. In fact, all the unifiers are instances of the substitution σ = `{X = Z, Y = king(ithaca, achaean)}`, which is the **most general unifier** or **MGU**.

Real Prolog systems display the unification of `T1` and `T2` in a slightly different way:

```
?- character(ulysses, Z, king(ithaca, achaean)) =
character(ulysses, X, Y).
X = _G123, Y = king(ithaca, achaean), Z = _G123
```

where `_Gxyz` are variable names internal to the Prolog system.

A.4.3 The Herbrand Unification Algorithm

The reference algorithm to unify terms is due to Herbrand (Herbrand 1930, Martelli and Montanari 1982). It takes the two terms to unify as input. The output is either a failure if terms do not unify or the MGU – σ.

The algorithm initializes the substitution to the empty set and pushes terms on a stack. The main loop consists in popping terms, comparing their functors, and pushing their arguments on the stack. When a variable is found, the corresponding substitution is added to σ (Sterling and Shapiro 1994, Deransart et al. 1996).

```
• Initialization step
  Initialize $\sigma$ to $\{\}$
  Initialize failure to false
  Push the equation $T_1 = T_2$ on the stack
• Loop
  repeat {
      pop $x = y$ from the stack
      if $x$ is a constant and $x == y$. Continue.
      else if $x$ is a variable and $x$ does not appear in $y$.
          Replace $x$ with $y$ in the stack and in $\sigma$. Add the substitution $\{x = y\}$ to
          $\sigma$.
      else if $x$ is a variable and $x == y$. Continue.
      else if $y$ is a variable and $x$ is not a variable.
          Push $y = x$ on the stack.
      else if $x$ and $y$ are compounds with $x = f(x_1, ..., x_n)$ and $y = f(y_1, ..., y_n)$.
          Push on the stack $x_i = y_i$ for $i$ ranging from 1 to $n$.
      else Set failure to true, and $\sigma$ to $\{\}$. Break.
  } until (stack $\neq \emptyset$)
```

A.4.4 Example

Let us exemplify the Herbrand algorithm with terms: `f(g(X, h(X, b)), Z)` and `f(g(a, Z), Y)`. We will use a two-way stack: one for the left term and one for the right term, and let us scan and push term arguments from right to left.

For the first iteration of the loop, x and y are compounds. After this iteration, the stack looks like:

Left term of the stack (x)		**Right term of the stack (y)**
`g(X, h(X, b))`	=	`g(a, Z)`
`Z`	=	`Y`

with the substitution σ = {}.

The second iteration pops the top terms of the left and right parts of the stack. The loop condition corresponds to compound terms again. The algorithm pushes the arguments of left and right terms on the stack:

Left term of the stack (x)		Right term of the stack (y)
X	=	a
h(X, b)	=	Z
Z	=	Y

with the substitution σ = {}.

The third iteration pops the equation `X = a`. The algorithm adds this substitution to σ and carries out the substitution in the stack:

Left term of the stack (x)		Right term of the stack (y)
h(X, b) $\sim$ h(a, b)	=	Z
Z	=	Y

with the substitution σ = `{X = a}`.

The next iteration pops `h(a, b) = Z`, swaps the left and right terms, and yields:

Left term of the stack (x)		Right term of the stack (y)
Z	=	h(a, b)
Z	=	Y

The fifth iteration pops `Z = h(a, b)` and yields:

Left term of the stack (x)		Right term of the stack (y)
Z $\sim$ h(a, b)	=	Y

with the substitution σ = {`X = a, Z = h(a, b)`}.

Finally, we get the MGU σ = {`X = a, Z = h(a, b), Y = h(a, b)`} that yields the unified term `f(g(a, h(a, b)), h(a, b))`.

A.4.5 The Occurs-Check

The Herbrand algorithm specifies that variables `X` or `Y` must not appear – occur – in the right or left member of the equation to be a successful substitution. The unification of `X` and `f(X)` should then fail because `f(X)` contains `X`.

However, most Prolog implementations do not check the occurrence of variables to keep the unification time linear on the size of the smallest of the terms being unified (Pereira 1984). Thus, the unification `X = f(X)` unfortunately succeeds resulting in a stack overflow. The term `f(X)` infinitely replaces `X` in σ, yielding `X = f(f(X))`, `f(f(f(X)))`, `f(f(f(f(X))))`, etc., until the memory is exhausted. It results into a system crash with many Prologs.

Although theoretically better, a unification algorithm that would implement an occurs-check is not necessary most of the time. An experienced programmer will not write unification equations with a potential occurs-check problem. That is why Prolog systems compromised the algorithm purity for speed. Should the occurs-check be necessary, Standard Prolog provides the `unify_with_occurs_check/2` built-in predicate:

```
?- unify_with_occurs_check(X, f(X)).
No

?- unify_with_occurs_check(X, f(a)).
X = f(a)
```

A.5 Resolution

A.5.1 Modus Ponens

The Prolog resolution algorithm is based on the *modus ponens* form of inference that stems from traditional logic. The idea is to use a general rule – the major premise – and a specific fact – the minor premise – like the famous:

All men are mortal
Socrates is a man

to conclude, in this case, that

Socrates is mortal

Table A.1 shows the modus ponens in the classical notation of predicate logic and in Prolog.

Table A.1. The modus ponens notation in formal logic and its Prolog equivalent.

	Formal notation	**Prolog notation**
Facts	α	`man('Socrates').`
Rules	$\underline{\alpha \Rightarrow \beta}$	`mortal(X) :- man(X).`
Conclusion	β	`mortal('Socrates').`

Prolog runs a reversed modus ponens. Using symbols in Table A.1, Prolog tries to prove that a query (β) is a consequence of the database content (α, $\alpha \Rightarrow \beta$). Using the major premise, it goes from β to α, and using the minor premise, from α to true. Such a sequence of goals is called a **derivation**. A derivation can be finite or infinite.

A.5.2 A Resolution Algorithm

Prolog uses a resolution algorithm to chain clauses mechanically and prove a query. This algorithm is generally derived from Robinson's resolution principle (1965), known as the SLD resolution. SLD stands for "linear resolution" with a "selection function" for "definite clauses" (Kowalski and Kuehner 1971). Here "definite clauses" are just another name for Prolog clauses.

The resolution takes a program – a set of clauses, rules, and facts – and a query `Q` as an input (Sterling and Shapiro 1994, Deransart et al. 1996). It considers a conjunction of current goals to prove, called the **resolvent**, that it initializes with `Q`. The resolution algorithm selects a goal from the resolvent and searches a clause in the database so that the head of the clause unifies with the goal. It replaces the goal with the body of that clause. The resolution loop replaces successively goals of the resolvent until they all reduce to true and the resolvent becomes empty. The output is then a success with a possible instantiation of the query goal `Q'`, or a failure if no rule unifies with the goal. In case of success, the final substitution, σ, is the composition of all the MGUs involved in the resolution restricted to the variables of `Q`. This type of derivation, which terminates when the resolvent is empty, is called a **refutation**.

```
• Initialization
  Initialize Resolvent to Q, the initial goal of the resolution algorithm.
  Initialize $\sigma$ to $\{\}$
  Initialize failure to false
• Loop with Resolvent = G$_1$, G$_2$, ..., G$_i$, ..., G$_m$
  while (Resolvent $\neq \emptyset$) {
      1. Select the goal G$_i$ $\in$ Resolvent;
      2. If G$_i$ == true, delete it and continue;
      3. Select the rule H :- B$_1$, ..., B$_n$ in the database such that G$_i$ and H
         unify with the MGU $\theta$. If there is no such a rule then set failure to
         true; break;
      4. Replace G$_i$ with B$_1$, ..., B$_n$ in Resolvent
         % Resolvent = G$_1$,...,G$_{i-1}$, B$_1$,...,B$_n$, G$_{i+1}$,..., G$_m$
      5. Apply $\theta$ to Resolvent and to Q;
      6. Compose $\sigma$ with $\theta$ to obtain the new current $\sigma$;
  }
```

Each goal in the resolvent – i.e., in the body of a rule – must be different from a variable. Otherwise, this goal must be instantiated to a nonvariable term before it is called. The `call/1` built-in predicate then executes it as in the rule:

```
daughter(X, Y) :-
  mother(Y, X), G = female(X), call(G).
```

where `call(G)` solves the goal `G` just as if it were `female(X)`. In fact, Prolog automatically inserts `call/1` predicates when it finds that a goal is a variable. `G` is thus exactly equivalent to `call(G)`, and the rule can be rewritten more concisely in:

```
daughter(X, Y) :-
  mother(Y, X), G = female(X), G.
```

A.5.3 Derivation Trees and Backtracking

The resolution algorithm does not tell us how to select a goal from the resolvent. It also does not tell how to select a clause in the program. In most cases, there is more than one choice. The selection order of goals is of no consequence because Prolog has to prove all of them anyway. In practice, Prolog considers the leftmost goal of the resolvent. The selection of the clause is more significant because some derivations lead to a failure although a query can be proved by other derivations. Let us show this with the program:

```
p(X) :- q(X), r(X).
q(a).
q(b).
r(b).
r(c).
```

and the query `?- p(X)`.

Let us compute the possible states of the resolvent along with the resolution's iteration count. The first resolvent (R1) is the query itself. The second resolvent (R2) is the body of `p(X)`: `q(X), r(X)`; there is no other choice. The third resolvent (R3) has two possible values because the leftmost subgoal `q(X)` can unify either with the facts `q(a)` or `q(b)`. Subsequently, according to the fact selected and the corresponding substitution, the derivation succeeds or fails (Fig. A.4).

$$\begin{tikzcd}
\text{R1:} & & \texttt{p(X)} \arrow[d] & \\
\text{R2:} & & \texttt{q(X), r(X)} \arrow[dl, "\sigma = \{\texttt{X = a}\}"'] \arrow[dr, "\sigma = \{\texttt{X = b}\}"] & \\
\text{R3:} & \texttt{q(a), r(a)} \arrow[d] & & \texttt{q(b), r(b)} \arrow[d] \\
\text{R4:} & \texttt{true, r(a)} & & \texttt{true, r(b)} \arrow[dd] \\
 & \texttt{failure} & & \\
\text{R5:} & & & \texttt{true} \\
 & & & \texttt{success}
\end{tikzcd}$$

Fig. A.4. The search tree and successive values of the resolvent.

The Prolog resolution can then be restated as a search, and the picture of successive states of the resolvent as a search tree. Now how does Prolog select a clause? When more than one is possible, Prolog could expand the resolvent as many times as there are clauses. This strategy would correspond to a breadth-first search. Although it gives all the solutions, this is not the one Prolog employs because would be unbearable in terms of memory.

Prolog uses a depth-first search strategy. It scans clauses from top to bottom and selects the first one to match the leftmost goal in the resolvent. This sometimes leads to a subsequent failure, as in our example, where the sequence of resolvents is first `p(X)`, then the conjunction `q(X), r(X)`, after that `q(a), r(a)`, and finally the goal `r(a)`, which is not in the database. Prolog uses a backtracking mechanism then.

During a derivation, Prolog keeps a record of backtrack points when there is a possible choice, that is, where more than one clause unifies with the current goal. When a derivation fails, Prolog backs up to the last point where it could select another clause, undoes the corresponding unification, and proceeds with the next possible clause. In our example, it corresponds to resolvent R2 with the second possible unification: `q(b)`. The resolvent R3 is then `q(b), r(b)`, which leads to a success. Backtracking explores all possible alternatives until a solution is found or it reaches a complete failure.

However, although the depth-first strategy enables us to explore most search trees, it is only an approximation of a complete resolution algorithm. In some cases, the search path is infinite, even when a solution exists. Consider the program:

```
p(X) :- p(X), q(X).
p(a).
q(a).
```

where the query `p(a)` does not succeed because of Prolog's order of rule selection. Fortunately, most of the time there is a workaround. Here it suffices to invert the order of the subgoals in the body of the rule.

A.6 Tracing and Debugging

Bugs are programming errors, that is, when a program does not do what we expect from it. To isolate and remove them, the programmer uses a **debugger**. A debugger enables programmers to trace the goal execution and unification step by step. It would certainly be preferable to write bug-free programs, but to err is human. And debugging remains, unfortunately, a frequent part of program development.

The Prolog debugger uses an execution model describing the control flow of a goal (Fig. A.5). It is pictured as a box representing the goal predicate with four ports, where:

- The Call port corresponds to the invocation of the goal.
- If the goal is satisfied, the execution comes out through the Exit port with a possible unification.
- If the goal fails, the execution exits through the Fail port.
- Finally, if a subsequent goal fails and Prolog backtracks to try another clause of the predicate, the execution re-enters the box through the Redo port.

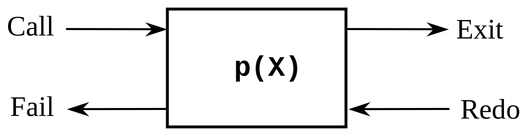


Fig. A.5. The execution model of Prolog.

The built-in predicate `trace/0` launches the debugger and `notrace/0` stops it. The debugger may have different commands according to the Prolog system you are using. Major ones are:

- `creep` to proceed through the execution ports. Simply type return to creep.
- `skip` to skip a goal giving the result without examining its subgoals. (type `s` to skip).
- `retry` starts the current goal again from an exit or redo port (type `r`).
- `fail` makes a current goal to fail (type `f`).
- `abort` to quit the debugger (type `a`).

Figure A.6 represents the rule `p(X) :- q(X), r(X)`, where the box corresponding to the head encloses a chain of subboxes picturing the conjunction of goals in the body. The debugger enters goal boxes using the `creep` command.

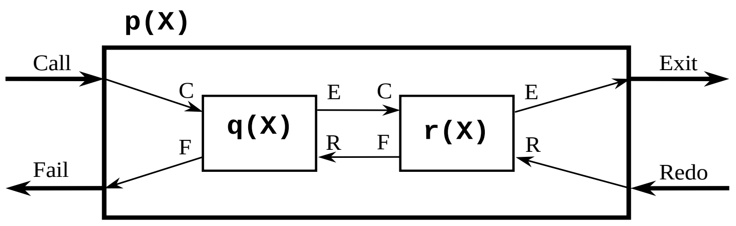


Fig. A.6. The execution box representing the rule `p(X) :- q(X), r(X)`.

As an example, let us trace the program:

```
p(X) :- q(X), r(X).
q(a).
q(b).
r(b).
r(c).
```

with the query `p(X)`.

```
?- trace.
Yes
?- p(X).
   Call:  (  7) p(_G106) ? creep
   Call:  (  8) q(_G106) ? creep
   Exit:  (  8) q(a) ? creep
   Call:  (  8) r(a) ? creep
   Fail:  (  8) r(a) ? creep
   Redo:  (  8) q(_G106) ? creep
   Exit:  (  8) q(b) ? creep
   Call:  (  8) r(b) ? creep
```

```
     Exit:  (  8) r(b) ? creep
     Exit:  (  7) p(b) ? creep
X = b
```

A.7 Cuts, Negation, and Related Predicates

A.7.1 Cuts

The cut predicate, written "`!`", is a device to prune some backtracking alternatives. It modifies the way Prolog explores goals and enables a programmer to control the execution of programs. When executed in the body of a clause, the cut always succeeds and removes backtracking points set before it in the current clause. Figure A.7 shows the execution model of the rule `p(X) :- q(X), !, r(X)` that contains a cut.

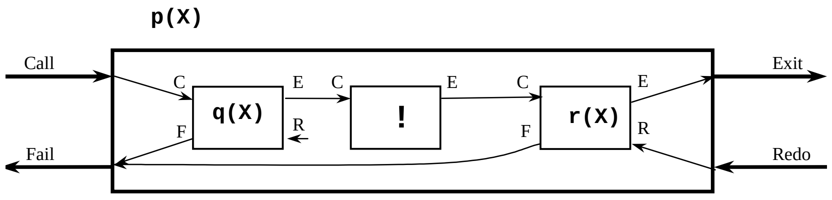


Fig. A.7. The execution box representing the rule `p(X) :- q(X), !, r(X).`

Let us suppose that a predicate `P` consists of three clauses:

P :- A_1, ..., A_i, !, A_{i+1}, ..., A_n.
P :- B_1, ..., B_m.
P :- C_1, ..., C_p.

Executing the cut in the first clause has the following consequences:

1. All other clauses of the predicate below the clause containing the cut are pruned. That is, here the two remaining clauses of `P` will not be tried.
2. All the goals to the left of the cut are also pruned. That is, A_1, ..., A_i will no longer be tried.
3. However, it will be possible to backtrack on goals to the right of the cut.

P :- ~~A_1, ..., A_i,~~ !, A_{i+1}, ..., A_n.
~~P :- B_1, ..., B_m.~~
~~P :- C_1, ..., C_p.~~

Cuts are intended to improve the speed and memory consumption of a program. However, wrongly placed cuts may discard some useful backtracking paths and solutions. Then, they may introduce vicious bugs that are often difficult to track. Therefore, cuts should be used carefully.

An acceptable use of cuts is to express determinism. Deterministic predicates always produce a definite solution; it is not necessary then to maintain backtracking possibilities. A simple example of it is given by the minimum of two numbers:

```
minimum(X, Y, X) :- X < Y.
minimum(X, Y, Y) :- X >= Y.
```

Once the comparison is done, there is no means to backtrack because both clauses are mutually exclusive. This can be expressed by adding two cuts:

```
minimum(X, Y, X) :- X < Y, !.
minimum(X, Y, Y) :- X >= Y, !.
```

Some programmers would rewrite `minimum/3` using a single cut:

```
minimum(X, Y, X) :- X < Y, !.
minimum(X, Y, Y).
```

The idea behind this is that once Prolog has compared `X` and `Y` in the first clause, it is not necessary to compare them again in the second one. Although the latter program may be more efficient in terms of speed, it is obscure. In the first version of `minimum/3`, cuts respect the logical meaning of the program and do not impair its legibility. Such cuts are called **green cuts**. The cut in the second `minimum/3` predicate is to avoid writing a condition explicitly. Such cuts are error-prone and are called **red cuts**. Sometimes red cuts are crucial to a program but when overused, they are a bad programming practice.

A.7.2 Negation

A logic program contains no negative information, only queries that can be proven or not. The Prolog built-in negation corresponds to a query failure: the program cannot prove the query. The negation symbol is written "`\+`" or `not` in older Prolog systems:

- If `G` succeeds then `\+ G` fails.
- If `G` fails then `\+ G` succeeds.

The Prolog negation is defined using a cut:

```
\+(P) :- P, !, fail.
\+(P) :- true.
```

where `fail/0` is a built-in predicate that always fails.

Most of the time, it is preferable to ensure that a negated goal is ground: all its variables are instantiated. Let us illustrate it with the somewhat odd rule:

```
mother(X, Y) :- \+ male(X), child(Y, X).
```

and facts:

```
child(telemachus, penelope).
male(ulysses).
male(telemachus).
```

The query

```
?- mother(X, Y).
```

fails because the subgoal `male(X)` is not ground and unifies with the fact `male(ulysses)`. If the subgoals are inverted:

```
mother(X, Y) :- child(Y, X), \+ male(X).
```

the term `child(Y, X)` unifies with the substitution `X = penelope` and `Y = telemachus`, and since `male(penelope)` is not in the database, the goal `mother(X, Y)` succeeds.

Predicates similar to "\+" include if-then and if-then-else constructs. If-then is expressed by the built-in `'->'/2` operator. Its syntax is

```
Condition -> Action
```

as in

```
print_if_parent(X, Y) :-
  (parent(X, Y) -> write(X), nl, write(Y), nl).

?- print_if_parent(X, Y).
penelope
telemachus

X = penelope, Y = telemachus
```

Just like negation, `'->'/2` is defined using a cut:

```
'->'(P, Q):- P, !, Q.
```

The if-then-else predicate is an extension of `'->'/2` with a second member to the right. Its syntax is

```
Condition -> Then ; Else
```

If `Condition` succeeds, `Then` is executed, otherwise `Else` is executed.

A.7.3 The `once/1` Predicate

The built-in predicate `once/1` also controls Prolog execution. `once(P)` executes `P` once and removes backtrack points from it. If `P` is a conjunction of goals as in the rule:

```
A :- B1, B2, once((B3, ..., Bi)), Bi+1, ..., Bn.
```

the backtracking path goes directly from $\texttt{B}_{i+1}$ to $\texttt{B}_2$, skipping $\texttt{B}_3$`, ..., `$\texttt{B}_i$. It is necessary to bracket the conjunction inside `once` twice because its arity is equal to one. A single level of brackets, as in `once(`$\texttt{B}_3$`, ..., `$\texttt{B}_i$`)`, would tell Prolog that `once/1` has an arity of `i-3`.

`once(Goal)` is defined as:

```
once(Goal) :- Goal, !.
```

A.8 Lists

Lists are data structures essential to many programs. A Prolog list is a sequence of an arbitrary number of terms separated by commas and enclosed within square brackets. For example:

- `[a]` is a list made of an atom.
- `[a, b]` is a list made of two atoms.
- `[a, X, father(X, telemachus)]` is a list made of an atom, a variable, and a compound term.
- `[[a, b], [[[father(X, telemachus)]]]]` is a list made of two sub-lists.
- `[]` is the atom representing the empty list.

Although it is not obvious from these examples, Prolog lists are compound terms and the square bracketed notation is only a shortcut. The list functor is a dot: "`./2`", and `[a, b]` is equivalent to the term `.(a,.(b,[]))`.

Computationally, lists are recursive structures. They consist of two parts: a head, the first element of a list, and a tail, the remaining list without its first element. The head and the tail correspond to the first and second argument of the Prolog list functor. Figure A.8 shows the term structure of the list `[a, b, c]`. The tail of a list is possibly empty as in `.(c,[]))`.

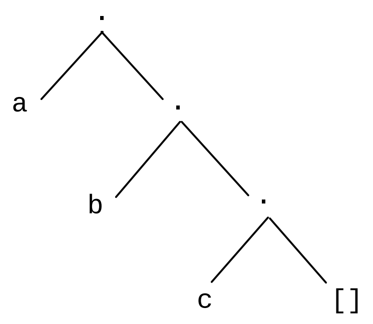


Fig. A.8. The term structure of the list `[a, b, c]`.

The notation "|" splits a list into its head and tail, and `[H | T]` is equivalent to `.(H, T)`. Splitting a list enables us to access any element of it and therefore it is a very frequent operation. Here are some examples of its use:

```
?- [a, b] = [H | T].
H = a, T = [b]

?- [a] = [H | T].
H = a, T = []

?- [a, [b]] = [H | T].
H = a, T = [[b]]

?- [a, b, c, d] = [X, Y | T].
X = a, Y = b, T = [c, d]

?- [[a, b, c], d, e] = [H | T].
H = [a, b, c], T = [d, e]
```

The empty list cannot be split:

```
?- [] = [H | T].
No
```

A.9 Some List-Handling Predicates

Many applications require extensive list processing. This section describes some useful predicates. Generally, Prolog systems provide a set of built-in list predicates. Consult your manual to see which ones; there is no use in reinventing the wheel.

A.9.1 The `member/2` Predicate

The `member/2` predicate checks whether an element is a member of a list:

```
?- member(a, [b, c, a]).
Yes

?- member(a, [c, d]).
No
```

`member/2` is defined as

```
member(X, [X | Y]).     % Termination case
member(X, [Y | YS]) :-  % Recursive case
  member(X, YS).
```

We could also use anonymous variables to improve legibility and rewrite `member/2` as

```
member(X, [X | _]).
member(X, [_ | YS]) :- member(X, YS).
```

`member/2` can be queried with variables to generate elements member of a list, as in:

```
?- member(X, [a, b, c]).
X = a ;
X = b ;
X = c ;
No
```

Or lists containing an element:

```
?- member(a, Z).
Z = [a | Y] ;
Z = [Y, a | X] ;
etc.
```

Finally, the query:

```
?- \+ member(X, L).
```

where `X` and `L` are ground variables, returns `Yes` if `member(X, L)` fails and `No` if it succeeds.

A.9.2 The `append/3` Predicate

The `append/3` predicate appends two lists and unifies the result to a third argument:

```
?- append([a, b, c], [d, e, f], [a, b, c, d, e, f]).
Yes

?- append([a, b], [c, d], [e, f]).
No

?- append([a, b], [c, d], L).
L = [a, b, c, d]

?- append(L, [c, d], [a, b, c, d]).
L = [a, b]

?- append(L1, L2, [a, b, c]).
L1 = [], L2 = [a, b, c] ;
L1 = [a], L2 = [b, c] ;
```

etc., with all the combinations.

`append/3` is defined as

```
append([], L, L).
append([X | XS], YS, [X | ZS]) :-
  append(XS, YS, ZS).
```

A.9.3 The `delete/3` Predicate

The `delete/3` predicate deletes a given element from a list. Its synopsis is: `delete(List, Element, ListWithoutElement)`. It is defined as:

```
delete([], _, []).
delete([E | List], E, ListWithoutE):-
  !,
  delete(List, E, ListWithoutE).
delete([H | List], E, [H | ListWithoutE]):-
  H \= E,
  !,
  delete(List, E, ListWithoutE).
```

The three clauses are mutually exclusive, and the cuts make it possible to omit the condition `H \= E` in the second rule. This improves the program efficiency but makes it less legible.

A.9.4 The `intersection/3` Predicate

The `intersection/3` predicate computes the intersection of two sets represented as lists: `intersection(InputSet1, InputSet2, Intersection)`.

```
?- intersection([a, b, c], [d, b, e, a], L).
L = [a, b]
```

`InputSet1` and `InputSet2` should be without duplicates; otherwise `intersection/3` approximates the intersection set relatively to the first argument:

```
?- intersection([a, b, c, a], [d, b, e, a], L).
L = [a, b, a]
```

The predicate is defined as:

```
% Termination case
intersection([], _, []).
% Head of L1 is in L2
intersection([X | L1], L2, [X | L3]) :-
  member(X, L2),
  !,
  intersection(L1, L2, L3).
% Head of L1 is not in L2
intersection([X | L1], L2, L3) :-
  \+ member(X, L2),
  !,
  intersection(L1, L2, L3).
```

As for `delete/3`, clauses of `intersection/3` are mutually exclusive, and the programmer can omit the condition `\+ member(X, L`$_2$`)` in the third clause.

A.9.5 The `reverse/2` Predicate

The `reverse/2` predicate reverses the elements of a list. There are two classic ways to define it. The first definition is straightforward but consumes much memory. It is often called the naïve reverse:

```
reverse([],[]).
reverse([X | XS], YS] :-
  reverse(XS,, RXS),
  append(RX, [X], Y).
```

A second solution improves the memory consumption. It uses a third argument as an accumulator.

```
reverse(X, Y) :-
  reverse(X, [], Y).

reverse([], YS, YS).
reverse([X | XS], Accu, YS):-
  reverse(XS, [X | Accu], YS).
```

A.9.6 The Mode of an Argument

The **mode** of an argument defines if it is typically an input (`+`) or an output (`-`). Inputs must be instantiated, while outputs are normally uninstantiated. Some predicates have multiple modes of use. We saw three modes for `append/3`:

- `append(+List1, +List2, +List3)`,
- `append(+List1, +List2, -List3)`, and
- `append(-List1, -List2, +List3)`.

A question mark "`?`" denotes that an argument can either be instantiated or not. Thus, the two first modes of `append/3` can be compacted into `append(+List1, +List2, ?List3)`. The actual mode of `append/3`, which describes all possibilities is, in fact, `append(?List1, ?List2, ?List3)`. Finally, "`@`" indicates that the argument is normally a compound term that shall remain unaltered.

It is a good programming practice to annotate predicates with their common modes of use.

A.10 Operators and Arithmetic

A.10.1 Operators

Prolog defines a set of prefix, infix, and postfix operators that includes the classical arithmetic symbols: "`+`", "`-`", "`*`", and "`/`". The Prolog interpreter considers operators as functors and transforms expressions into terms. Thus, `2 * 3 + 4 * 2` is equivalent to `+(*(2, 3), *(4, 2))`.

The mapping of operators onto terms is governed by rules of priority and classes of associativity:

- The priority of an operator is an integer ranging from 1 to 1200. It enables us to determine recursively the principal functor of a term. Higher-priority operators will be higher in the tree representing a term.
- The associativity determines the bracketing of term `A op B op C`:
  1. If `op` is left-associative, the term is read `(A op B) op C`;
  2. If `op` is right-associative, the term is read `A op (B op C)`.

Prolog defines an operator by its name, its **specifier**, and its priority. The specifier is a mnemonic to denote the operator class of associativity and whether it is infixed, prefixed, or postfixed (Table A.2).

Table A.2. Operator specifiers.

Operator	Nonassociative	Right-associative	Left-associative
Infix	*xfx*	*xfy*	*yfx*
Prefix	*fx*	*fy*	–
Postfix	*xf*	–	*yf*

Table A.3 shows the priority and specifier of predefined operators in Standard Prolog.
It is possible to declare new operators using the directive:

```
:- op(+Priority, +Specifier, +Name).
```

A.10.2 Arithmetic Operations

The evaluation of an arithmetic expression uses the `is/2` built-in operator. `is/2` computes the value of the `Expression` to the right of it and unifies it with `Value`:

```
?- Value is Expression.
```

where `Expression` must be computable. Let us exemplify it. Recall first that "`=`" does not evaluate the arithmetic expression:

Table A.3. Priority and specifier of operators in Standard Prolog.

Priority	Specifier	Operators
1200	xfx	:- -->
1200	fx	:- ?-
1100	xfy	;
1050	xfy	->
1000	xfy	','
900	fy	\+
700	xfx	= \=
700	xfx	== \== @< @=< @> @>=
700	xfx	=..
700	xfx	is =:= =\= < =< > >=
550	xfy	:
500	yfx	+ - # /\ \/
400	yfx	* / // rem mod << >>
200	xfx	**
200	xfy	^
200	fy	+ - \

```
?- X = 1 + 1 + 1.
X = 1 + 1 + 1 (or X = +(+(1, 1), 1)).
```

To get a value, it is necessary to use `is`

```
?- X = 1 + 1 + 1, Y is X.
X = 1 + 1 + 1, Y = 3.
```

If the arithmetic expression is not valid, `is/2` returns an error, as in

```
?- X is 1 + 1 + a.
Error
```

because `a` is not a number, or as in

```
?- X is 1 + 1 + Z.
Error
```

because `Z` is not instantiated to a number. But

```
?- Z = 2, X is 1 + 1 + Z.
Z = 2, X = 4
```

is correct because `Z` has a numerical value when `X` is evaluated.

A.10.3 Comparison Operators

Comparison operators process arithmetic and literal expressions. They evaluate arithmetic expressions to the left and to the right of the operator before comparing them, for example:

```
?- 1 + 2 < 3 + 4.
Yes
```

Comparison operators for literal expressions rank terms according to their lexical order, for example:

```
?- a @< b.
Yes
```

Standard Prolog defines a lexical ordering of terms that is based on the ASCII value of characters and other considerations. Table A.4 shows a list of comparison operators for arithmetic and literal expressions.

Table A.4. Comparison operators.

	Arithmetic comparison	Literal term comparison
Equality operator	`=:=`	`==`
Inequality operator	`=\=`	`\==`
Inferior	`<`	`@<`
Inferior or equal	`=<`	`@=<`
Superior	`>`	`@>`
Superior or equal	`>=`	`@>=`

It is a common mistake of beginners to confuse the arithmetic comparison (`=:=`), literal comparison (`==`), and even sometimes unification (`=`). Unification is a logical operation that finds two substitutions to render two terms identical; an arithmetic comparison computes the numerical values of the left and right expressions and compares their resulting value; a term comparison compares literal values of terms but does not perform any operation on them. Here are some examples:

```
?- 1 + 2 =:= 2 + 1.
Yes
?- 1 + 2 = 2 + 1.
No
?- 1 + 2 = 1 + 2.
Yes
?- 1 + X = 1 + 2.
X = 2
?- 1 + X =:= 1 + 2.
Error
```

```
?- 1 + 2 == 1 + 2.
Yes
?- 1 + 2 == 2 + 1.
No
?- 1 + X == 1 + 2.
No
?- 1 + a == 1 + a.
Yes
```

A.10.4 Lists and Arithmetic: The `length/2` Predicate

The `length/2` predicate determines the length of a list

```
?- length([a, b, c], 3).
Yes

?- length([a, [a, b], c], N).
N = 3
```

`length(+List, ?N)` traverses the list `List` and increments a counter `N`. Its definition in Prolog is:

```
length([],0).
length([X | XS], N) :-
  length(XS, N1),
  N is N1 + 1.
```

The order of subgoals in the rule is significant because `N1` has no value until Prolog has traversed the whole list. This value is computed as Prolog pops the recursive calls from the stack. Should subgoals be inverted, the computation of the length would generate an error telling that `N1` is not a number.

A.10.5 Lists and Comparison: The `quicksort/2` Predicate

The `quicksort/2` predicate sorts the elements of a list `[H | T]`. It first selects an arbitrary element from the list to sort, here the head, `H`. It splits the list into two sublists containing the elements smaller than this arbitrary element and the elements greater. `Quicksort` then sorts both sublists recursively and appends them once they are sorted. In this program, the `before/2` predicate compares the list elements using the `@</2` literal operator.

```
% quicksort(+InputList, -SortedList)

quicksort([], []) :- !.
quicksort([H | T], LSorted) :-
  split(H, T, LSmall, LBig),
  quicksort(LSmall, LSmallSorted),
  quicksort(LBig, LBigSorted),
  append(LSmallSorted, [H | LBigSorted], LSorted).

split(X, [Y | L], [Y | LSmall], LBig) :-
  before(Y, X),
  !,
  split(X, L, LSmall, LBig).
split(X, [Y | L], LSmall, [Y | LBig]) :-
  !,
    split(X, L, LSmall, LBig).
split(_, [], [], []) :- !.

before(X, Y) :- X @< Y.
```

A.11 Some Other Built-in Predicates

The set of built-in predicates may vary according to Prolog implementations. Here is a list common to many Prologs. Consult your reference manual to have the complete list.

A.11.1 Type Predicates

The type predicates check the type of a term. Their mode of use is `type_predicate(?Term)`.

- `integer/1`: Is the argument an integer?

```
?- integer(3).
Yes

?- integer(X).
No
```

- `number/1`: Is the argument a number?

```
?- number(3.14).
Yes
```

- `float/1`: Is the argument a floating-point number?
- `atom/1`: Is the argument an atom?

```
?- atom(abc).
Yes

?- atom(3).
No
```

- `atomic/1`: Is the argument an atomic value, i.e., a number or an atom?
- `var/1`: Is the argument a variable?

```
?- var(X).
Yes

?- X = f(Z), var(X).
No
```

- `nonvar/1`: The opposite of `var/1`.

```
?- nonvar(X).
No
```

- `compound/1`: Is the argument a compound term?

```
?- compound(X).
No

?- compound(f(X, Y)).
Yes
```

- `ground/1`: Is the argument a ground term?

```
?- ground(f(a, b)).
Yes

?- ground(f(a, Y)).
No
```

A.11.2 Term Manipulation Predicates

The term manipulation predicates enable us to access and modify elements of compound terms.

- The built-in predicate `functor(+Term, ?Functor, ?Arity)` gets the principal functor of a term and its arity.

```
?- functor(father(ulysses, telemachus), F, A).
F = father, A = 2
```

`functor` also returns the most general term given a functor name and an arity. `Functor` and `Arity` must then be instantiated: `functor(-Term, +Functor, +Arity)`

```
?- functor(T, father, 2).
T = father(X, Y)
```

- The predicate `arg(+N, +Term, ?X)` unifies `X` to the argument of rank `N` in `Term`.

```
?- arg(1, father(ulysses, telemachus), X).
X = ulysses
```

- The operator `Term =.. List`, also known as the *univ* predicate, transforms a term into a list.

```
?- father(ulysses, telemachus) =.. L.
L = [father, ulysses, telemachus]

?- T =.. [a, b, c].
T = a(b, c)
```

Univ has two modes of use: `+Term =.. ?List`, or `-Term =.. +List`.

- The predicate `name(?Atom, ?List)` transforms an atom into a list of ASCII codes.

  ```
  ?- name(abc, L).
  L = [97, 98, 99]

  ?- name(A, [97, 98, 99]).
  A = abc
  ```

  Standard Prolog provides means to encode strings more naturally using double quotes. Thus

  ```
  ?- "abc" = L.
  L = [97, 98, 99]
  ```

A.12 Handling Run-Time Errors and Exceptions

Standard Prolog features a mechanism to handle run-time errors. An error or exception occurs when the execution cannot be completed normally either successfully or by a failure. Examples of exceptions include division by zero, the attempt to evaluate arithmetically nonnumerical values with `is/2`, and calling a noninstantiated variable in the body of a rule:

```
?- X is 1/0.
ERROR: //2: Arithmetic evaluation error: zero_divisor

?- X is 1 + Y.
ERROR: Arguments are not sufficiently instantiated

?- X.
ERROR: Arguments are not sufficiently instantiated
```

In the normal course of a program, such faulty clauses generate run-time errors and stop the execution. The programmer can also trap these errors and recover from them using the `catch/3` built-in predicate.

`catch(+Goal, ?Catcher, ?Recover)` executes `Goal` and behaves like `call/1` if no error occurs. If an error is raised and unifies with `Catcher`, `catch/3` proceeds with `Recover` and continues the execution.

Standard Prolog defines catchers of built-in predicates under the form of the term `error(ErrorTerm, Information)`, where `ErrorTerm` is a standard description of the error and `Information` depends on the implementation. The query:

```
?- catch((X is 1 + Y), Error, (write(Error),nl,fail)).
error(instantiation_error,
```

```
  context(system: (is)/2, _GXyz))

No
```

attempts to execute `X is Y + 1`, raises an error, and executes the recover goal, which prints the error and fails. The constant `instantiation_error` is part of the set of error cases defined by Standard Prolog.

Built-in predicates execute a `throw/1` to raise exceptions when they detect an error. The `throw` predicate immediately goes back to a calling `catch/3`. If there is no such `catch`, by default, the execution is stopped and the control is transferred to the user.

User-defined predicates can also make use of `throw(+Exception)` to throw an error, as in:

```
throw_error :- throw(error(error_condition,context)).
```

The corresponding error can be caught as in the query:

```
?- catch(throw_error, Error, (write(Error),nl,fail)).
error(error_condition, context)

No
```

A.13 Dynamically Accessing and Updating the Database

A.13.1 Accessing a Clause: The `clause/2` Predicate

The built-in predicate `clause(+Head, ?Body)` returns the body of a clause whose head unifies with `Head`. Let us illustrate this with the program:

```
hero(ulysses).
heroin(penelope).

daughter(X, Y) :-
  mother(Y, X),
  female(X).
daughter(X, Y) :-
  father(Y, X),
  female(X).
```

and the query:

```
?- clause(daughter(X, Y), B).
B = (mother(Y, X), female(X));
B = (father(Y, X), female(X));
No

?- clause(heroin(X), B).
X = penelope, B = true.
```

A.13.2 Dynamic and Static Predicates

The built-in predicates `asserta/1`, `assertz/1`, `retract/1`, and `abolish/1` add or remove clauses – rules and facts – during the execution of a program. They allow to update the database – and hence to modify the program – dynamically.

A major difference between Prolog implementations is whether the system interprets the program or compiles it. Roughly, an interpreter does not change the format of rules and facts to run them. A compiler translates clauses into a machine-dependent code or into more efficient instructions (Maier and Warren 1988). A compiled program runs much faster then.

Compiling occurs once at load time, and the resulting code is no longer modifiable during execution. To run properly, the Prolog engine must be told which predicates are alterable at run-time – the **dynamic** predicates – and which ones will remain unchanged – the **static** predicates. Prolog compiles static predicates and runs dynamic predicates using an interpreter.

A predicate is static by default. Dynamic predicates must either be declared using the `dynamic/1` directive or be entirely created by assertions at run time. In the latter case, the first assertion of a clause declares automatically the new predicate to be dynamic. The directive specifying that a predicate is dynamic precedes all its clauses, if any. For example, the program:

```
:- dynamic parent/2, male/1.
...
parent(X, Y) :-
...
male(xy).
...
```

declares that `parent/2` and `male/1` clauses may be added or removed at run time.

The predicates `asserta/1`, `assertz/1`, `retract/1`, and `abolish/1` can modify clauses of dynamic predicates only. Adding or removing a clause for a static predicate raises an error condition.

A.13.3 Adding a Clause: The `asserta/1` and `assertz/1` Predicates

The predicate `asserta(+P)` adds the clause `P` to the database. `P` is inserted just before the other clauses of the same predicate. As we have seen before, the predicate corresponding to the clause `P` must be dynamic: declared using the `dynamic/1` directive or entirely asserted at run time.

```
                               % State of the database
                               % Before assertion
                               % hero(ulysses).
                               % hero(hector).
?- asserta(hero(achilles)).
                               % State of the database
                               % After assertion
                               % hero(achilles).
                               % hero(ulysses).
                               % hero(hector).
```

The predicate `assertz/1` also adds a new clause, but as the last one of the procedure this time.

Adding rules is similar. It requires double parentheses, as in

```
asserta((P :- B, C, D)).
```

However, it is never advised to assert rules. Modifying rules while running a program is rarely useful and may introduce nasty bugs.

Novice Prolog programmers may try to communicate the results of a procedure by asserting facts to the database. This is not a good practice because it hides what is the real output of a predicate. Results, especially intermediate results, should be passed along from one procedure to another using arguments. Assertions should only reflect a permanent change in the program state.

A.13.4 Removing Clauses: The `retract/1` and `abolish/2` Predicates

The built-in predicates `retract/1` and `abolish/1` remove clauses of a dynamic predicate. `retract(+P)` retracts clause `P` from the database.

```
                             % State of the database
                             % Before removal
                             % hero(ulysses).
                             % hero(achilles).
                             % hero(hector).
?- retract(hero(hector)).
                             % State of the database
                             % After
                             % hero(ulysses).
                             % hero(achilles).
?- retract(hero(X)).
X = ulysses ;
X = achilles ;
No
?- hero(X).
No
```

The predicate `abolish(+Predicate/Arity)` removes all clauses of `Predicate` with arity `Arity` from the database.

A.13.5 Handling Unknown Predicates

When a static predicate is called and is not in the database, it is often a bug. A frequent cause is due to wrong typing as, for example, `parnet(X, Y)` instead of `parent(X, Y)`, where `n` and `e` are twiddled. For this reason, by default, Prolog raises an error in the case of such a call.

An effect of `dynamic/1` is to declare a predicate to the Prolog engine. Such a predicate 'exists' then, even if it has no clauses. A call to a dynamic predicate that has no clauses in the database is not considered as an error. It fails, simply and silently.

The Prolog engine behavior to calls to unknown predicates can be modified using the `unknown/2` directive:

```
:- unknown(-OldValue, +NewValue).
```

where `OldValue` and `NewValue` can be:

- `warning` – A call to an unknown predicate issues a warning and fails.
- `error` – A call to an unknown predicate raises an error. As we saw, this is the default value.
- `fail` – A call to an unknown predicate fails silently.

A Prolog flag also defines this behavior. It can be set by `set_prolog_flag/2`:

```
?- set_prolog_flag(+FlagName, +NewValue).
```

where `FlagName` is set to `unknown` and possible values are `error`, `warning`, or `fail`. The current flag status is obtained by `current_prolog_flag/2`:

```
?- current_prolog_flag(+FlagName, ?Value).
```

A.14 All-Solutions Predicates

The second-order predicates `findall/3`, `bagof/3`, and `setof/3` return all the solutions to a given query. The predicate `findall` is the basic form of all-solutions predicates, while `bagof` and `setof` are more elaborate. We exemplify them with the database:

```
character(ulysses, iliad).
character(hector, iliad).
character(achilles, iliad).
character(ulysses, odyssey).
character(penelope, odyssey).
character(telemachus, odyssey).
```

`findall(+Variable, +Goal, ?Solution)` unifies `Solution` with the list of all the possible values of `Variable` when querying `Goal`.

```
?- findall(X, character(X, iliad), B).
B = [ulysses, hector, achilles]

?- findall(X, character(X, Y), B).
B = [ulysses, hector, achilles, ulysses, penelope,
telemachus]
```

The predicate `bagof(+Variable, +Goal, ?Solution)` is similar to `findall/3`, except that it backtracks on the free variables of `Goal`:

```
?- bagof(X, character(X, iliad), Bag).
Bag = [ulysses, hector, achilles]

?- bagof(X, character(X, Y), Bag).
Bag =[ ulysses, hector, achilles], Y = iliad ;
Bag = [ulysses, penelope, telemachus], Y = odyssey ;
No.
```

Variables in `Goal` are not considered free if they are existentially quantified. The existential quantifier uses the infix operator "ˆ". Let `X` be a variable in `Goal`. `XˆGoal` means that there exists `X` such that `Goal` is true. `bagof/3` does not backtrack on it. For example:

```
?- bagof(X, Y^character(X, Y), Bag).
Bag = [ulysses, hector, achilles, ulysses,
penelope, telemachus]

?- bagof(X, Y^(character(X, Y), female(X)), Bag).
Bag = [penelope]
```

The predicate `setof(+Variable, +Goal, ?Solution)` does the same thing as `bagof/3`, except that the `Solution` list is sorted and duplicates are removed from it:

```
?- setof(X, Y^character(X, Y), Bag).
Bag = [achilles, hector, penelope, telemachus,
ulysses]
```

A.15 Fundamental Search Algorithms

Many problems in logic can be represented using a graph or a tree, where finding a solution corresponds to searching a path going from an initial state to a goal state. The search procedure starts from an initial node, checks whether the current node meets a goal condition, and if not, goes to a next node. The transition from one node to a next one is carried out using a successor predicate, and the solution is the sequence of nodes traversed to reach the goal. In the context of search, the graph is also called the **state space**.

In this section, we will review some fundamental search strategies and as an application example, we will try to find our way through the labyrinth shown in Fig. A.9. As we saw, Prolog has an embedded search mechanism that can be used with little adaptation to implement other algorithms. It will provide us with the Ariadne's thread to remember our way in the maze with minimal coding efforts.

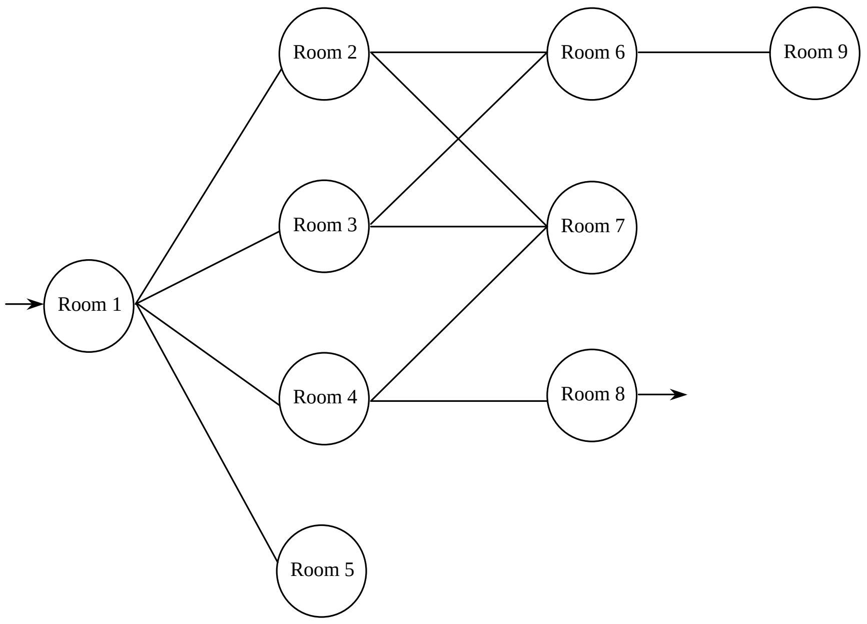


Fig. A.9. The graph representing the labyrinth.

A.15.1 Representing the Graph

We use a successor predicate `s(X, Y)` to represent the graph, where `Y` is the successor of `X`. For the labyrinth, the `s/2` predicate describes the immediate links from one room to another. The links between rooms are:

```
link(r1, r2). link(r1, r3). link(r1, r4). link(r1,
r5). link(r2, r6). link(r2, r7). link(r3, r6).
link(r3, r7). link(r4, r7). link(r4, r8). link(r6,
r9).
```

Since links can be traversed both ways, the `s/2` predicate is:

```
s(X, Y) :- link(X, Y).
s(X, Y) :- link(Y, X).
```

The goal is expressed as:

```
goal(X) :- minotaur(X).
```

where

```
minotaur(r8).
```

Finally, we could associate a cost to the link, for instance, to take into account its length. The predicate would then be:

```
s(X, Y, Cost).
```

A.15.2 Depth-First Search

A depth-first search is just the application of the Prolog resolution strategy. It explores the state space by traversing a sequence of successors to the initial node until it finds a goal. The search goes down the graph until it reaches a node without successor. It then backtracks from the bottom to the last node that has successors.

Searching a path in a labyrinth is then very similar to other programs we have written before. It consists of a first rule to describe the goal condition and second recursive one to find a successor node when the condition is not met. The `depth_first_search(+Node, -Path)` predicate uses the initial node as input and returns the path to reach the goal:

```
%% depth_first_search(+Node, -Path)
depth_first_search(Node, [Node]) :-
  goal(Node).
depth_first_search(Node, [Node | Path]) :-
  s(Node, Node1),
  depth_first_search(Node1, Path).
```

This short program does not work, however, because the path could include infinite cycles: Room 2 to Room 6 to Room 2 to Room 6…To prevent them, we need to remember the current path in an accumulator variable and to avoid the successors of the current node that are already members of the path. We use a `depth_first_search/3` auxiliary predicate, and the new program is:

```
%% depth_first_search(+Node, -Path)
depth_first_search(Node, Path) :-
  depth_first_search(Node, [], Path).

%% depth_first_search(+Node, +CurrentPath,-FinalPath)
depth_first_search(Node, Path, [Node | Path]) :-
  goal(Node).
```

```
depth_first_search(Node, Path, FinalPath) :-
  s(Node, Node1),
  \+ member(Node1, Path),
  depth_first_search(Node1, [Node | Path],FinalPath).
```

The result of the search is:

```
?- depth_first_search(r1, L).
L = [r8, r4, r7, r3, r6, r2, r1] ;
L = [r8, r4, r7, r2, r1] ;
L = [r8, r4, r7, r2, r6, r3, r1] ;
L = [r8, r4, r7, r3, r1] ;
L = [r8, r4, r1] ;
No
?-
```

A.15.3 Breadth-First Search

The breadth-first search explores the paths in parallel. It starts with the first node, all the successors of the first node, all the successors of the successors, and so on, until it finds a solution.

If the list `[Node | Path]` describes a path to a node, the search needs to expand all the successors of `Node`. It generates the corresponding paths as lists. There are as many lists as there are successors to `Node`. The search then sets the successors as the heads of these lists. This is done compactly using the `bagof/3` predicate:

```
expand([Node | Path], ExpandedPaths) :-
  bagof(
    [Node1, Node | Path],
    (s(Node, Node1), \+ member(Node1, Path)),
    ExpandedPaths).
```

As with the depth-first search, the breadth-first search consists of two rules. The first rule describes the goal condition. It extracts the first path from the list and checks whether the head node is a goal. The second rule implements the recursion. It expands the first path – the head of the list – into a list of paths that go one level deeper in the graph and appends them to the end of the other paths. The `breadth_first_search(+Node, -Path)` predicate uses the initial node as input and returns the path to reach the goal. The program needs to start with a list of lists, and it uses the auxiliary predicate `bf_search_aux/2`.

```
%% breadth_first_search(+Node, -Path)
breadth_first_search(Node, Path) :-
  bf_search_aux([[Node]], Path).

bf_search_aux([[Node | Path] | _], [Node | Path]) :-
  goal(Node).
```

```
bf_search_aux([CurrentPath | NextPaths],
    FinalPath) :-
  expand(CurrentPath, ExpandedPaths),
  append(NextPaths, ExpandedPaths, NewPaths),
  bf_search_aux(NewPaths, FinalPath).
```

The program is not completely correct, however, because `expand/2` can fail and make the whole search fail. A failure of `expand/2` means that the search cannot go further in this path and it has found no goal node in it. We can remove the path from the list then. To reflect this, we must add a second rule to `expand/2` that sets the path to the empty list and prevents the first rule from backtracking:

```
expand([Node | Path], ExpandedPaths) :-
  bagof(
    [Node1, Node | Path],
    (s(Node, Node1), \+ member(Node1, Path)),
    ExpandedPaths),
  !.
expand(Path, []).
```

The result of the search is:

```
?- breadth_first_search(r1, L).
L = [r8, r4, r1] ;
L = [r8, r4, r7, r2, r1] ;
L = [r8, r4, r7, r3, r1] ;
L = [r8, r4, r7, r3, r6, r2, r1] ;
L = [r8, r4, r7, r2, r6, r3, r1] ;
No
?-
```

The breadth-first search strategy guarantees that it will find the shortest path to the solution. A disadvantage is that it must store and maintain all exploration paths in parallel. This requires a huge memory, even for a limited search depth.

A.15.4 A* Search

The A* search is a variation and an optimization of the breadth-first search. Instead of expanding the first path of the list, it uses heuristics to select a better candidate. While searching the graph, A* associates a value to paths it traverses. This value is a function f of the node being traversed. $f(n)$ at node n is the sum of two terms $f(n) = g(n) + h(n)$, where $g(n)$ is the length of the path used to reach node n and $h(n)$ is the estimate of the remaining length to reach the goal node. From a given node, A* ranks the possible subsequent nodes minimizing $f(n)$. It then explores "best nodes" first and thus avoids a blind searching.

The main difficulty of the A* search is to find a suitable h function. Its presentation is outside the scope of this appendix. Russell and Norvig (2003) examine search strategies in detail. Bratko (2001) describes an implementation of A* in Prolog.

A.16 Input/Output

The first Prolog systems had only primitive input/output facilities. Standard Prolog defines a complete new set of predicates. They represent a major change in the Prolog language, and although they are more flexible they are not universally accepted yet. This section introduces both sets of predicates. It outlines Standard Prolog input/output predicates and predicates conforming to the older tradition of Edinburgh Prolog. Most input/output predicates are deterministic, that is, they give no alternative solutions upon backtracking.

A.16.1 Reading and Writing Characters with Edinburgh Prolog

In Edinburgh Prolog, reading characters from the keyboard and writing to the screen is carried out using `get0/1` and `put/1`. Both predicates process characters using their ASCII codes. `get0/1` unifies with -1 when it reaches the end of a file. Here are some examples of use:

```
?- get0(X).
a ?

X = 97

?- put(65).
a

?- get0(X).
^D

X = -1
```

A.16.2 Reading and Writing Terms with Edinburgh Prolog

The built-in predicates `read/1` and `write/1` read and write terms from the current input and output streams. `read(?Term)` reads one term:

```
?- read(X).
character(ulysses, odyssey).

X = character(ulysses, odyssey)
```

where the input term must be terminated by a period. When reaching the end of a file, `X` unifies with the build-in atom `end_of_file`:

```
?- read(X).
^D
X = end_of_file
```

Writing terms is similar. `write(+Term)` writes one term to the current output stream and `nl/0` prints a new line:

```
?- T = character(ulysses, odyssey), write(T), nl.
character(ulysses, odyssey)

T = character(ulysses, odyssey)
?-
```

A.16.3 Opening and Closing Files with Edinburgh Prolog

Prolog input and output predicates normally write on the screen – the standard output – and read from the keyboard – the standard input. The predicates `see/1` and `tell/1` redirect the input and output so that a program can read or write any file.

`see/1` and `tell/1` open a file for reading and for writing. Then input/output predicates such as `get0/1`, `read/1` or `put/1`, `write/1` are redirected to the current open file. Several files may be open at the same time. The program switches between open files using `see/1` or `tell/1` until they are closed. `seen/0` and `told/0` close the open input and the open output, respectively, and return to the standard input/output, that is, to the keyboard and the screen. Let us show this with an example.

`see(in_file),`	Opens `in_file` as the current input stream.
`see(user),`	The current stream becomes the user – the keyboard.
`see(in_file),`	`in_file` becomes the current input stream again with the reading the position it had before.
`seen,`	Closes the current input stream. The current stream becomes the keyboard.
`seeing(IN_STREAM),`	`IN_STREAM` unifies with the current input stream.
`tell(out_file),`	Opens `out_file` as the current output stream (creates a new file or empties a previously existing file).
`telling(OUT_STREAM),`	`OUT_STREAM` unifies with the current output stream.
`tell(user),`	The current output stream becomes the user – the screen.
`told.`	Closes the current output stream. The current output stream becomes the user.

Here is a short program to read a file:

```
read_file(FileName, CodeList) :-
  see(FileName),
  read_list(CodeList),
  seen.

read_list([C | L]) :-
  get0(C),
  C =\= -1, % end of file
  !,
  read_list(L).
read_list([]).
```

A.16.4 Reading and Writing Characters with Standard Prolog

Standard Prolog uses streams to read and write characters. A stream roughly corresponds to an open file. Streams are divided into output streams or sinks, and input streams or sources. By default, there are two current open streams: the standard input stream, which is usually the keyboard, and the standard output stream, the screen. Other streams are opened and closed using `open/4`, `open/3`, `close/1`, and `close/2`.

The predicates to read and write a character are `get_char/1`, `get_char/2`, `put_char/1`, and `put_char/2`:

- `get_char(?Char)` unifies `Char` with the next character of the current input stream.
- `get_char(+Stream, ?Char)` unifies `Char` with the next character of the open input stream `Stream`. `get_char/1` and `get_char/2` predicates unify with `end_of_file` when they reach the end of a file.
- `put_char(+Char)` writes `Char` to the current output stream.
- `put_char(+Stream, ?Char)` writes `Char` to the open output `Stream`.
- `nl/0` and `nl(+Stream)` write a new line to the current output stream or to `Stream`.

Here is a short example:

```
?- get_char(X).
a ?

X = a

?- put_char(a).
a

?- get_char(X).
^D

X = end_of_file
```

Instead of reading and writing characters, we may want to read or write their numeric code, ASCII or Unicode, as with Edinburgh's `get0/1`. The corresponding Standard Prolog predicates are `get_code/1`, `get_code/2`, `put_code/1`, and `put_code/2`.

The predicates `get_char` and `get_code` read a character or a code, remove it from the input stream, and move to the next character. Sometimes it is useful to read a character without removing it. The predicates `peek_char` and `peek_code` do just that. They unify with the current character but stay at the same position and leave the character in the stream.

A.16.5 Reading and Writing Terms with Standard Prolog

The Standard Prolog predicates `read/1` and `write/1` are identical to those of Edinburgh Prolog:

- `read(?Term)` reads one term from the current input stream.
- `write(+Term)` writes a term to the current output stream.

`read/2` and `write/2` read and write terms from and to a file:

- `read(+Stream, ?Term)` reads a term from `Stream`.
- `write(+Stream, ?Term)` writes a term to `Stream`.

The predicates `read_term` and `write_term` read and write terms with a list of options, either to the current input/output, `read_term/2` and `write_term/2`, or to a file, `read_term/3` and `write_term/3`. The options make it possible to adjust the printing format, for instance. They may depend on the implementation and the operating system. Consult your manual to have the complete list. The predicates `read` and `write` are equivalent to `read_term` and `write_term` with an empty list of options.

A.16.6 Opening and Closing Files with Standard Prolog

The predicates to open and close a stream are `open/4`, `open/3`, `close/1`, and `close/2`:

- `open(+SourceSink, +Mode, -Stream)` opens the file `SourceSink` in an input or output `Mode`. The `Mode` value is one of `read`, `write`, `append`, or `update`. `Stream` unifies with the opened stream and is used for the subsequent input or output operations.
- `open(+SourceSink, +Mode, -Stream, +Options)` opens the file with a list of options. `open/3` is equivalent to `open/4` with an empty list of options. Consult your manual to have the complete list.
- `close(+Stream)` closes the stream `Stream`.
- `close(+Stream, +Options)` closes the stream `Stream` with a list of options. `close/1` is equivalent to `close/2` with an empty list of options.

Here is a short program to read a file with Standard Prolog predicates:

```
read_file(FileName, CharList) :-
  open(FileName, read, Stream),
  read_list(Stream, CharList),
  close(Stream).

read_list(Stream, [C | L]) :-
  get_char(Stream, C),
  C \== end_of_file, % end of file
  !,
  read_list(Stream, L).
read_list(_, []).
```

Other useful predicates include `current_input/1`, `current_output/1`, `set_input/1`, and `set_output/1`:

- `current_input(?Stream)` unifies `Stream` with the current input stream.
- `current_output(?Stream)` unifies `Stream` with the current output.
- `set_input(+Stream)` sets `Stream` to be the current input stream.
- `set_output(+Stream)` sets `Stream` to be the current output stream.

A.16.7 Writing Loops

Programmers sometimes wonder how to write iterative loops in Prolog, especially with input/output to read or to write a sequence of terms. This is normally done with a recursive rule, as to read a file. Counting numbers down to 0 takes the form:

```
countdown(X) :-
  number(X),
  X < 0.
countdown(X):-
  number(X),
  X >= 0,
  write(X), nl,
  NX is X - 1,
  countdown(NX).
```

For example,

```
?- countdown(4).
4
3
2
1
0
?-
```

In some other cases, backtracking using the `repeat/0` built-in predicate can substitute a loop. The `repeat/0` definition is:

```
repeat.
repeat :- repeat.
```

`repeat` never fails and when inserted as a subgoal, any subsequent backtracking goes back to it and the sequence of subgoals to its right gets executed again. So, a sequence of subgoals can be executed any number of times until a condition is satisfied. The `read_write/1` predicate below reads and writes a sequence of atoms until the atom `end` is encountered. It takes the form of a repetition (`repeat`) of reading a term `X` using `read/1`, writing it (`write/1`), and a final condition (`X == end`). It corresponds to the rule:

```
read_write :-
  repeat,
  read(X),
  write(X), nl,
  X == end,
  !.
```

A.17 Developing Prolog Programs

A.17.1 Presentation Style

Programs are normally written once and then are possibly read and modified several times. A major concern of the programmer should be to write clear and legible code. It helps enormously with the maintenance and debugging of programs.

Before programming, it is essential first to have a good formulation and decomposition of the problem. The program construction should then reflect the logical structure of the solution. Although this statement may seem obvious, its implementation is difficult in practice. Clarity in a program structure is rarely attained from the first time. First attempts are rarely optimal but Prolog enables an incremental development where parts of the solution can be improved gradually.

A key to the good construction of a program is to name things properly. Cryptic predicates or variable names, such as `syntproc`, `def_code`, `X`, `Ynn`, and so on, should be banned. It is not rare that one starts with a predicate name and changes it in the course of the development to reflect a better description of the solution.

Since Prolog code is compact, the code of a clause should be short to remain easy to understand, especially with recursive programs. If necessary, the programmer should decompose a clause into smaller subclauses. Cuts and asserts should be kept to a minimum because they impair the declarativeness of a program. However, these are general rules that sometimes are difficult to respect when speed matters most.

Before its code definition, a predicate should be described in comments together with argument types and modes:

```
% predicate(+Arg1, +Arg2, -Arg3).
% Does this and that
% Arg1: list, Arg2: atom, Arg3: integer.
```

Clauses of a same predicate must be grouped together, even if some Prologs permit clauses to be disjoined. The layout of clauses should also be clear and adopt common rules of typography. Insert a space after commas or dots, for instance. The rule

```
pred1 :- pred2(c,d),e,f.
```

must be rejected because of sticking commas and obfuscated predicate names. Goals must be indented with tabulations, and there should be one single goal per line. Then

```
A :-
  B,
  C,
  D.
```

should be preferred to

```
A :- B, C, D.
```

except when the body consists of a single goal. The rule

```
A :- B.
```

is also acceptable.

A.17.2 Improving Programs

Once a program is written, it is generally possible to enhance it. This section introduces three techniques to improve program speed: goal ordering, memo functions, and tail recursion.

Order of Goals. Ordering goals is meaningful for the efficiency of a program because Prolog tries them from left to right. The idea is to reduce the search space as much as possible from the first goals. If predicate $\mathtt{p}_1$ has 1000 solutions in 1 s and $\mathtt{p}_2$ has 1 solution taking 1000 hours to compute, avoid conjunction:

```
p1(X), p2(X).
```

A better ordering is:

```
p2(X), p1(X).
```

Lemmas or Memo Functions. Lemmas are used to improve the program speed. They are often exemplified with Fibonacci series. Fibonacci imagined around year 1200 how to estimate a population of rabbits, knowing that:

- A rabbit couple gives birth to another rabbit couple, one male and one female, each month (one month of gestation).
- A rabbit couple reproduces from the second month.
- Rabbits are immortal.

We can predict the number of rabbit couples at month n as a function of the number of rabbit couples at month $n - 1$ and $n - 2$:

$$rabbit(n) = rabbit(n - 1) + rabbit(n - 2)$$

A first implementation is straightforward from the formula:

```
fibonacci(1, 1).
fibonacci(2, 1).
fibonacci(M, N) :-
  M > 2,
  M1 is M - 1, fibonacci(M1, N1),
  M2 is M - 2, fibonacci(M2, N2),
  N is N1 + N2.
```

However, this program has an expensive double recursion and the same value can be recomputed several times. A better solution is to store Fibonacci values in the database using `asserta/1`. So an improved version is

```
fibonacci(1, 1).
fibonacci(2, 1).
fibonacci(M, N) :-
  M > 2,
  M1 is M - 1, fibonacci(M1, N1),
  M2 is M - 2, fibonacci(M2, N2),
  N is N1 + N2,
  asserta(fibonacci(M, N)).
```

The rule is then tried only if the value is not in the database.
The generic form of the lemma is:

```
lemma(P):-
  P,
  asserta((P :- !)).
```

with “`!`” to avoid backtracking.

Tail Recursion. A tail recursion is a recursion where the recursive call is the last subgoal of the last rule, as in

```
f(X) :- fact(X).
f(X) :- g(X, Y), f(Y).
```

Recursion is generally very demanding in terms of memory, which grows with the number of recursive calls. A tail recursion is a special case that the interpreter can transform into an iteration. Most Prolog systems recognize and optimize it. They execute a tail-recursive predicate with a constant memory size.

It is therefore significant not to invert clauses of the previous program, as in

```
f(X) :- g(X, Y), f(Y).
f(X) :- fact(X).
```

which is not tail recursive.

It is sometimes possible to transform recursive predicates into a tail recursion equivalent, adding a variable as for `length/2`:

```
length(List, Length) :-
  length(List, 0, Length).

length([], N, N).
length([X | L], N1, N) :-
  N2 is N1 + 1,
  length(L, N2, N).
```

It is also sometimes possible to force a tail recursion using a cut, for example,

```
f(X) :- g(X, Y), !, f(Y).
f(X) :- fact(X).
```

Exercises

A.1. Describe a fragment of your family using Prolog facts.

A.2. Using the model of `parent/2` and `ancestor/2`, write rules describing family relationships.

A.3. Write a program to describe routes between cities. Use a `connect/2` predicate to describe direct links between cities as facts, for example, `connect(paris, london)`, `connect(london, edinburgh)`, etc., and write the `route/2` recursive predicate that finds a path between cities.

A.4. Unify the following pairs:

```
f(g(A, B), a) = f(C, A).
f(X, g(a, b)) = f(g(Z), g(Z, X)).
f(X, g(a, b)) = f(g(Z), g(Z, Y)).
```

A.5. Trace the `son/2` program.

A.6. What is the effect of the query

```
?- f(X, X).
```

given the database:

```
f(X, Y) :- !, g(X), h(Y).
g(a).
g(b).
h(b).
```

A.7. What is the effect of the query

```
?- f(X, X).
```

given the database:

```
f(X, Y) :- g(X), !, h(Y).
g(a).
g(b).
h(b).
```

A.8. What is the effect of the query

```
?- f(X, X).
```

given the database:

```
f(X, Y) :- g(X), h(Y), !.
g(a).
g(b).
h(b).
```

A.9. What is the effect of the query

```
?- \+  f(X,  X).
```

given the databases of the three previous exercises (Exercises A.6–A.8)? Provide three answers.

A.10. Write the `last(?List, ?Element)` predicate that succeeds if `Element` is the last element of the list.

A.11. Write the `nth(?Nth, ?List, ?Element)` predicate that succeeds if `Element` is the `Nth` element of the list.

A.12. Write the `maximum(+List, ?Element)` predicate that succeeds if `Element` is the greatest of the list.

A.13. Write the `flatten/2` predicate that flattens a list, i.e., removes nested lists:

```
?- flatten([a, [a, b, c], [[[d]]]], L).
L = [a, a, b, c, d]
```

A.14. Write the `subset(+Set1, +Set2)` predicate that succeeds if `Set1` is a subset of `Set2`.

A.15. Write the `subtract(+Set1, +Set2, ?Set3)` predicate that unifies `Set3` with the subtraction of `Set2` from `Set1`.

A.16. Write the `union(+Set1, +Set2, ?Set3)` predicate that unifies `Set3` with the union of `Set2` and `Set1`. `Set1` and `Set2` are lists without duplicates.

A.17. Write a program that transforms the lowercase characters of a file into their uppercase equivalent. The program should process accented characters, for example, *é* will be mapped to *É*.

A.18. Implement A* in Prolog.